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THE RANK INPUT METHOD AND PROBABILITY VARIATION GUIDES.(U)
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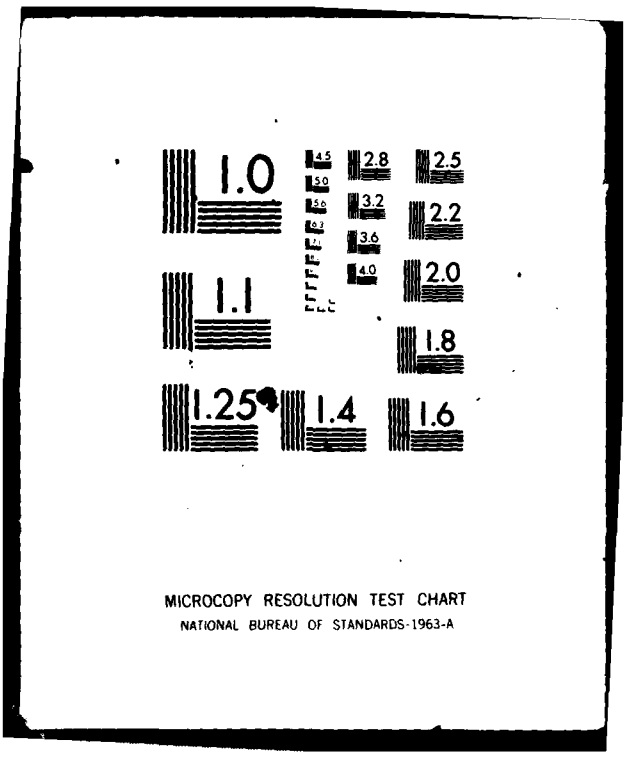

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THE RANK INPUT METHOD AND PROBABILITY VARIATION GUIDES

Albert R. Boehm, Major, USAF

July 1980

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UNITED STATES AIR FORCE
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The technical note has been reviewed and is approved for publication.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The rank input method allows a forecaster's subjective estimate to be quantified into a probability forecast. The forecaster's estimate can be a rank input, a probability of a single category, or a categorical forecast. With the rank input the forecaster ranks the synoptic situation--very bad to very good--in relation to the element to be forecast, e.g., surface visibility. The transnormalized regression probability model is then used to calculate the probability of the specific event. Probability of a single category can be converted to probabilities for one or more different categories. A categorical (Cont'd)

20. ABSTRACT (Cont'd)

forecast can be converted to probability forecasts. A validation during REFORGER 78 concluded that the method shows promise and that forecasters were able to produce a large number of probability forecasts with a few simple rankings of the synoptic situation. Probability variation guides are tables giving forecast probability values for various inputs. Plotted on a simple graph, all values for a given skill and climatology fall along a single curve in probability space. These curves make certain decision analysis theorems much simpler in form.

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THE RANK INPUT METHOD AND PROBABILITY VARIATION GUIDES

1. INTRODUCTION

The author has presented numerous training sessions on making subjective probability forecasts. From this experience he finds a forcaster can make good reliable subjective forecasts. Given standard weather information, all that is additionally needed is the basic knowledge of probability, practice training, the time to formulate a forecast, and prompt verification feedback. However, urgent military requirements sometimes leave little time for training for new locations or categories. Also, it is straightforward to subjectively forecast for two categories, but it is time consuming to determine probabilities for many categories. For example, the TEAM SPIRIT exercise in Korea in 1977 required probabilities for six ceiling categories. The rank input method (RIM) and probability variation guides (PVG) have been developed to solve this problem.

The RIM and PVG assume that a forecaster with training and experience can generate the degree of certainty that a synoptic situation favors a specific event. Ordinarily, additional training and in particular verification feedback, is necessary to convert the degree of certainty into a reliable probability forecast. The basic idea of the RIM is to use an objective procedure to convert the subjective degree of certainty into a reliable probability forecast of an event. A PVG is a table of RIM output for manual use.

The forecaster's input can be a rank input, a single category input, or a categorical forecast. These inputs are effectively scaled by the climatological frequency of the event and the forecaster skill. To understand the equations involved, some knowledge of the Transnormalized Regression Probability Forecast Model (TRP) is needed.

2. TRANSNORMALIZED REGRESSION PROBABILITY

The TRP model is a statistical model that can be used to produce probability forecasts in an empirical fashion such as regression estimate of event probabilities (REEP) and multiple discriminant analysis (MDA). However, TRP differs in two respects. First, TRP distinguishes between direct predictors which are generated by nature and have inherent distribution and marginal predictors such as location or verification time which are selected by man and have no inherent distribution. The term marginal comes from the fact that these predictors adjust the distribution of the direct predictors which are considered marginal distributions of the multivariate normal.

The second major difference is the analytical nature of the TRP model. This analytical nature allows for a priori verification, forecast distributions, and modeling of correlation and climatology. The TRP model consists of three basic parts: transnormalization, correlation, and multivariate normal probability. Details of the TRP model are in Boehm (1976).

2.1 Transnormalization

Transnormalization is the process of transforming a variable so that it is normally distributed. Variables that have been transnormalized are called equivalent normal deviates (END) and are designated with a quote (') above the variable. A transnormalization procedure can be developed from climatology rather than the small concurrent predictor/predictand data base usually available. Having each variable normally distributed is a necessary (but not sufficient) condition for a multivariate normal distribution. The assumption of multivariate normality, given that each variable is individually normally distributed, is the basic and only assumption of the TRP model.

2.2 Correlation

The Pearson product moment coefficient of correlation is frequently used of calculate correlation between two variables; however, this procedure can give a highly biased estimate when one or both of the variables have been categorized. The mutivariate normal conditional probability equation requires the correlation between the continuous underlying variables before they have been categorized, not the correlation corresponding to the linear least squares fit between categorized variables. Pearson developed many other formulas to estimate the correlation between underlying variables given only the categorized variables. Tetrachoric correlation is used for correlating binary against binary. However, beware of several simple formulas that give rough estmates: a more exact estimate requires a series expansion. Biserial correlation is used when one variate is continuous and the other binary. The original formula developed by Pearson (and given in most statistical texts) is unstable; an interactive scheme has been developed by Tate (1955).

2.3 Multinormal Conditional Probability

The multinormal conditional probability of the predictand (Y) being less than a threshold ($\ddot{y}$) given the m predictors $x_1, x_2, \ldots, x_m$ is

$$Pr(Y \leq \ddot{y} | \ddot{x}_1, \ddot{x}_2, \ldots, \ddot{x}_m) = \Phi[(\ddot{y} - RN)/(1 - R^2)^{1/2}] \quad (1)$$

where N equals $(a_1x_1 + a_2x_2 + \ldots + a_mx_m)$ a_i are regression coefficients, and R is the correlation. The cumulative normal distribution, $\Phi(u)$, can be approximated (absolute error less than 0.0004) by Muench's (Touart, 1973) formula

$$\Phi(u) \doteq [1 + \text{Tanh}(0.8u + 0.035u^3)]/2 \quad (2)$$

Gringorten (1978) extended the equation for multiple predictands.

Given the assumption of multinormality, N is normally distributed with a mean of zero and a variance of one. Furthermore, the normality of N is quite robust since the sum of many random variables will tend to be normally distributed even if the individual variables are not normally distributed. The fact that the distribution of N is known is exploited in the rank input method.

3. RANK INPUT METHOD

Subjective input may be a rank input, a single probability (to produce multiple probabilities), or a categorical input. Each of these inputs is transformed to the equivalent value of N and then Equation (1) is used to produce probabilities for specific categories by changing $\ddot{y}$, the climatological value of the predictand. The method requires the climatology of the predictand and the forecaster's skill measured by the correlation R. For ceiling/visibility forecasts, R is approximately

$$R = 0.98^T \qquad (T \text{ is in hours}) \quad (3)$$

This model was fitted with data from Touart (1973) and tested with several thousand 3-, 6-, 12-, and 24-hour Air Weather Service Forecasts.

3.1 The Rank Input

With this procedure, the forecaster first ranks the synoptic situation with respect to the element being forecast. For example, with a warm front moving into the area of interest, the forecaster may decide that this is the 90 percent worst situation with respect to low ceilings. Here the term synoptic situation is taken to include the forecast position of synoptic features as well as the current position. Indeed, the forecaster needs to include all information (e.g., output from an objective forecast study) in his determination of the ranking. Note that during a long period, including normal weather fluctuation, rank inputs should be uniform. This is, the number of times the forecaster selects a rank 0-10 percent should equal (within sampling error) the number between 10-20 percent, or 20-30 percent, etc.

The rank input (w) is expressed as a fraction, e.g., 0.01 worst to 0.99 best. This fraction is converted to an END via the inverse cumulative normal which can be approximated (less than 0.05 error for w from 0.001 to 0.999) by a formula developed by Joiner and Rosenblatt (1971):

$$\ddot{w} \doteq [w^{0.14} - (1 - w)^{0.14}]\, 4.91 \quad (4)$$

The N in Equation (1) is set to minus $\ddot{w}$ which makes N the END of the cumulative best versus the cumulative worst. What is worst or best depends on a particular application. Thus, a double check on the sign of N is wise. An example of RIM using the rank input is given in Table 1.

Table 1. Example of a Probability Variation Guide Using the Rank Input Method. Rank printed here as percentage was converted to an END by Eq. (4), then Eq. (1) was used to calculate probabilities. Correlation was obtained from Eq. (3). Climatology of the four categories was: 0.03, 0.105, 0.207, and 0.685.

6-HOUR SCOTT AFB CEILING FORECAST
VALID DECEMBER 1800Z

RANK INPUT	0-200'	200'-1000'	1000'-3000'	3000' ABV
1	18	81	1	0
2	7	89	4	0
5	1	80	18	1
10	0	54	42	4
20	0	20	58	22
30	0	6	48	47
40	0	2	29	69
50	0	0	15	85
60	0	0	6	94
70	0	0	2	99
80	0	0	0	100
90	0	0	0	100
95	0	0	0	100
98	0	0	0	100
99	0	0	0	100

3.2 The Single Probability Input

Forecast probabilities for multiple ranked categories can be obtained from the forecast probability of a single category by solving for N in Equation (1)

$$N = [\ddot{y} - \ddot{P}\,(1-R^2)^{1/2}]/R \qquad (5)$$

where $\ddot{y}$ is the END of the climatological probability of the input category, and $\ddot{P}$ is the forecast probability of the input category. The calculated N is then used in Equation (1) to produce forecast probability for each of the other categories by substituting the appropriate y. An example of a RIM using a single probability input is given in Table 2.

Table 2. Example of a Probability Variation Guide Using a Single Category Input. Correlation is from Eq. (3). Climatology values are: input 0.3, predictands 0.01, 0.05, 0.1, 0.3, 0.54.

6-HOUR PROBABILITY (%) FORECAST
USING TYPICAL CEILING CATEGORIES

INPUT PROBABILITIY BLO 3000	0-200	200-1000	1000-2000	2000-5000	5000 & ABV
01	00	00	00	04	96
02	00	00	00	07	93
05	00	00	00	14	86
10	00	00	01	23	76
20	00	00	02	37	60
30	00	00	05	47	48
40	00	01	08	54	37
50	00	01	12	59	28
60	00	02	17	60	20
70	00	04	24	59	13
80	00	08	31	53	08
90	00	17	40	40	03
95	01	27	42	28	01
98	03	40	40	17	00
99	06	48	35	11	00

3.3 The Categorical Forecast Input

A categorical forecast (e.g., a 400-ft ceiling) can be converted to a probability forecast provided distribution of the categorical input is known. Two forecast distributions often occur.

One is that the categorical forecast is distributed the same as the observations. For example, Somerville, Watkins, and Daley (1978) found that observed sky cover could be fitted with a Johnson's Bounded distribution

$$\overset{\text{u}}{P} = A + B \ln [n/(1-n)] \qquad (6)$$

where $\overset{\text{u}}{P}$ is the END of the climatological distribution. A and B are parameters depending on location, time of day, and time of year, and n is the fraction of sky cover. If the forecast sky cover and observation distributions are the same, then the N in Equation (1) can be equal to P from Equation (6) given the categorical forecast of the n. Table 3 is an example of a RIM using a categorical forecast input.

The second way that categorical forecasts are likely to be distributed is that a categorical forecast is the result of a root-mean-square estimate. In this case, its variance will be less than the variance of the observed distribution by a fraction equal to the correlation squared. The procedure is to inflate the categorical predictor by dividing the deviation from the mean by the value of correlation and then using the above result.

Table 3. An Example of a Probability Variation Guide Using a Categorical Input. Climatology from Eq. (6) with A = 0.339, B = 0.217. CLR defined as less than 0.05 coverage; OVC greater than 0.95 coverage. The mid-point, e.g., 1/2 PR(CLR), was used as the predictor probability value for CLR and OVC.

PROBABILITY OF BEDFORD 0600 SKY COVER
USING A CATEGORICAL 30-HOUR FORECAST

SKY FCST	CLR	SCT	BRN	OVC
CLR	40	29	20	10
1/10	26	29	26	19
2/10	23	28	28	22
3/10	20	27	28	24
4/10	19	20	29	26
5/10	17	25	29	28
6/10	13	22	30	35
9/10	10	21	30	39
OVC	04	12	25	58

4. PROBABILITY VARIATION GUIDES

The RIM can be implemented on a real-time computer system or even on one of the smaller programmable calculators. However, if neither of these are available, tables of probability forecasts called Probability Variation Guides can be produced. Each location, forecast element, forecast projection, and verifying time requires a separate table. The tables may use the rank input (Table 1), single category (Table 2), or categorical input (Table 3).

The tables are also valuable for training forecasters and for use by planners who need to see the expectancy of various probability values.

5. COMPLETE FORECAST DISTRIBUTIONS

When the climatology of a variable can be expressed as a differentiable function, it is possible to generate complete forecast distributions with N as the only variable. To obtain such a result, the $\overset{\text{u}}{y}$ in Equation (1) is replaced with a climatological function and the right-hand side of Equation (1) is differentiated with respect to the predictand using Leibniz's theorem for differentiation of an integral.

Consider, for example, the climatological distribution of sky cover as given by Equation (6). Substituting Equation (6) into Equation (1) and differentiating gives

$$f(n) = \frac{B\phi\{(A+B\ \ln[n/(1-n)] - RN)/(1-R^2)^{1/2}\}}{(1-r^2)^{1/2}\ n(1-n)} \tag{7}$$

where ϕ is the ordinate of the standard normal density, and f(n) is the forecast probability density for a given fraction of sky cover (n) varying only with the single parameter N. For correlation equal to 0, the climatological probability density results

$$f(n) = \frac{B}{n(1-n)}\ \phi\ \{A+B\ \ln[n/(1-n)]\} \tag{8}$$

Typical curves of Equations (7) and (8) are shown in Figure 1.

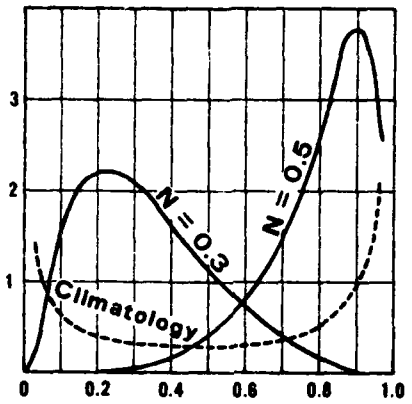

Figure 1. Fraction of Cloud Cover for Bedford, November 0600. Climatology from Eq. (8) with A = 0.339, B = 0.217. Forecast densities are from Eq. (7) with R = 0.98.

6. RANK INPUT AS A FORECASTER MULTIPLIER

Current operational forecasts are being made for the most part using a man-machine mix (Snellman 1977). The computer is used to compute large-scale weather changes and specific forecasts through objective methods. This guidance is passed to the forecaster who, using experience, "tunes" the information before passing it to the customer. Many computer programs are being developed to provide the forecaster with even more timely and varied guidance. The system works and is in widespread use. However, Snellman points out some danger signs. Forecasters are operating more as communicators and less as meteorologists. The system encourages the forecasters to follow guidelines blindly. The author has observed situations where the volume of guidance has inundated the forecaster who then has no time to properly assimilate the information, let alone determine an optimal tailored forecast. Therefore, this type of system could be termed a bottleneck system--with the forecaster as the bottleneck. The computer is driving the man rather than vice versa. Given this state of affairs a completely automated system then seems to be very practical.

Consider the entire man-machine mix. Computers have capability of errorless high-speed computation, storage, and formatting. But people have initiative, flexability, and ability to see the total picture. It appears we are not using the most effective combination of these qualities. An alternate system is to give the forecaster general synotic information and let him or her input into the computer. Let the computer build bulletins at proper times, add local climatology, and answer standard query response questions. In this setup the forecaster becomes the driver of the system and his capability has multiplied. The Rank Input Method provides an algorithm for such a system which is described in the following section.

6.1 The Probability Window

The concept of a probability window is based on the premise that for the longer forecast times, only the large-scale weather patterns can be forecast. However, a given larger scale weather pattern can provide detailed small-scale forecasts if detailed small-scale climatology is available. Therefore, the changable part of the forecast, i.e., the part that carries the forecast information, can be represented on a large-scale grid with relatively few numbers. The large-scale forecast can be interpolated to any point within the window. At that point the large-scale value is combined with point climatology to produce a point forecast. A description of a probability window using an objective input is given in Boehm (1977).

6.2 REFORGER 78

REFORGER is an annual exercise that tests the ability of US forces to support NATO commitments. In addition to standard weather support, probability forecasts for visibility and combined ceiling visibility categories were required in 1978. These forecasts were for a rectangular 13 X 10 mesogrid in central Germany with the southwest corner at 49°51'N. 8°20'E using a 10' east-west and 7' north-south grid spacing. The four categories were: (1) ceiling greater than or equal to 100 feet and visibility greater than or equal to 900 meters; (2) visibility greater than or equal to 1500 meters; (3) visibility greater than or equal to 3000 meters; and (4) ceiling greater than or equal 2000 feet and visibility greater than or equal to 800 meters. Forecast projections were 12, 18, 24, 30, 36, 48, and 60 hours. Separate subjective rankings for ceiling and visibility were input at four macroscale grid points located near the corners of the window. Thus, at a given projection eight (4 input points X 2 CIG VSBY) input rankings produced 520 (130 grids points X 4 categories) probability forecasts.

The procedure was verified 1 August to 30 September 1978 at four stations in Germany: Frankfurt AB (EDDF), Maurice Rose AAF (EDEN), Fulda AAF (EDEX), and Kitzingen AAF (EDIN). Brier scores for Frankfurt AB, 12-hour forecast, category 3 were RIM 0.30, climatology 0.37, sample climatology is the frequency of the event during the verification period, sample climatology 0.29. This relation held for all stations and forecast projections: the forecast showed a definite imporvement over climatology but sample climatology did even better. Keep in mind sample climatology is only completely known after the fact. Further analyst showed that ceiling and visibility was much higher than average. Frankfurt AB had category 4 93 percent of the time on the 12-hour forecast projection compared to the long-term climatology of 78 percent. A feedback system showing the accumulated ranking would have failed to correct for this run of good weather.

Major Frank Globakar (1978) who evaluated the method concluded:

> Capt (now Maj) Boehm's model shows promise. The forecaster did not have to be concerned with specific weather categories and was able to produce a large number of probability forecasts with a few simple rankings of the of the forecast synoptic situaiton. The method showed more skill than either persistence or climatology which are the usual benchmarks of skill.

7. PROBABILITY AND TRIANGULAR COORDINATES

Probabilities for three categories are often plotted on triangular coordinate graph paper since any point in the triangle presents a set of three probabilities that add up to 1. The idea can be extended to more than three categories. For four categories a three-sided equilateral pyramid is used. Here we will stay with three categories but keep in mind the result can be applied to a higher number of categories.

7.1 TRP In Triangular Coordinates

For the internal category of ranked categories (category 2 of 3) the TRP equation is

$$P_2 = Pr(y_1 \leq Y \leq y_2) = \Phi[(y_2-RN)/(1-R^2)^{1/2}] - \Phi[(y_1-RN)/(1-R^2)^{1/2}] \quad (9)$$

When P_1, P_2, and P_3 are plotted in triangular coordinates an interesting curve results. Most surprising is that all points fall on a single curved line, not an area. The line runs from $P_1 = 1$ to $P_3 = 1$, is symmetrical with respect to P_1 and P_3, and P_2 reaches a maximum value when $P_1 = P_3$. The higher the correlation and climatological probability of category 2, the closer the line approaches $P_2 = 1$. See Figures 2 and 3. The max P_2 is

$$\text{Max } P_2 = 2\,\Phi[(\bar{y}_2-\bar{y}_1)/[2(1-R^2)^{1/2}]] - 1 \quad (10)$$

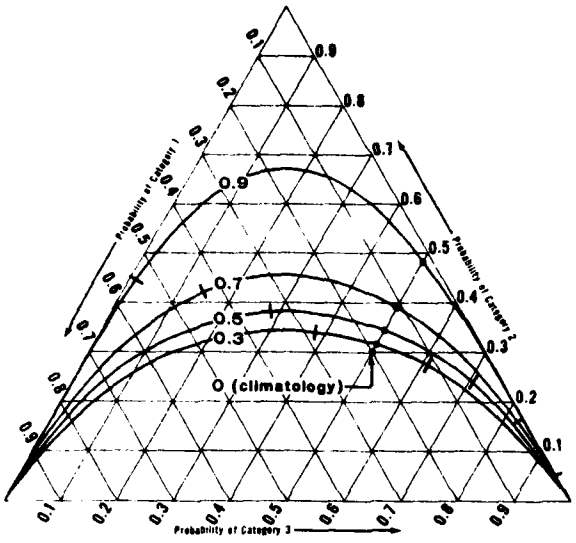

Figure 2. TRP Probability Values on Triangular Coordinate Paper. All curves use climatology of category (1) = 0.2, (2) = 0.3, and (3) = 0.5. Circles on the lines indicate N = 0, i.e., half of the forecasts will be to the left along the curve, the other half to the right. A dash across the curve indicates N = ±1, i.e., about 68 percent of the forecasts will lie between the two dashes.

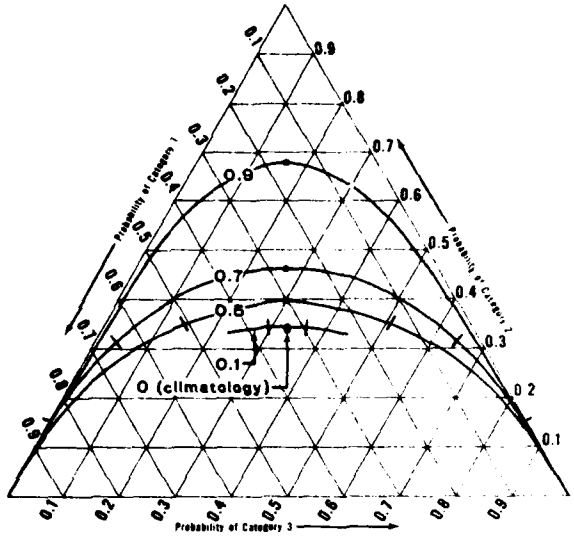

Figure 3. Same as Figure 1 except climatology of all categories is one-third. Forecast line for correlation of 0.3 is drawn only to N = ±3.

Proof: maximum difference between the right-hand terms of Equation (9) occurs with arguments centered on 0 since the slope is maximum at 0 and the absolute value monotonically decreases away from 0. To be centered about 0 requires the first argument to equal minus the second argument. Solve this equation for N and substitute into Equation (9). Then use $\Phi(-u) = 1 - \Phi(u)$.

7.2 Categorical Forecast Decision Rules

Inspection of the TRP Equations (1) and (9) shows that any point on a given curve is a function of the single parameter N. Keeping in mind the method is limited to ranked categories, any decision rule, in principle, can be rewritten with single parameter N or the equivalent vector (not matrix!) of critical probabilities of cummulative categories.

Since N is normally distributed, the expected frequency that forecasts will fall in any decision region can be calculated. For example, suppose the requirement is to have the number of times the end category is categorically forecast equal to its observed frequency. The critical probability P_c for this requirement is obtained by setting $N = \bar{y}$ in Equation (1) giving

$$Pc\ [Pr(OB) = Pr(FCST)] = \Phi[\bar{y}(1-R)/(1-R^2)^{1/2}] \tag{11}$$

Any scoring system can be evaluated with a given set of critical probabilities by simulating forecasts by varying N which is normally distributed. For two categories the expected frequency that a forecast will be successful (S) for a given critical probability P_c can be obtained by integration of the bivariate normal

$$S = \Phi\{[\bar{P}c(1-R^2)^{1/2}-\bar{y}]/R,\ \bar{y},\ R\} \tag{12}$$

This equation has been used to prepare tables (Boehm, 1977) for various critical probabilities as an aid to decision makers to subjectively determine a critical probability by weighing forecast hits against misses.

8. CONCLUSIONS

The Rank Input Method and Probability Variation Guides offer means for multiplying the effectiveness of a weather forecaster. They allow the forecaster to drive the computer system instead of the computer driving the forecaster. The single forecast index of the TRP model is usefull in many decision analysis problems and has many potential applications.

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Appendix
GENERAL FORM OF ERROR DISTRIBUTIONS

Let U be an observed variable, e.g., visibility, sky cover, etc. Let u be a specific value of U, e.g., three miles visibility. Let y be a climatological probability that U is less than or equal to u. Let y be the equivalent normal deviate of y.

If the forecast probability (P) is given by (1), then the change in probability for a change in u, i.e., the forecast density (dP/du) can be obtained by differentiating (1) with respect to u via Leibniz's theorem or differentiation of an intergral:

$$\frac{dp}{du} = \frac{\Phi\left[\overset{"}{y} - R\overset{"}{x})/(1 - R^2)^{1/2}\right]}{(1 - R^2)^{1/2}} \qquad \frac{d\overset{"}{y}}{du} \tag{A1}$$

When $\overset{"}{y}$ is given directly as a function of u, e.g., the lognormal distribution, then the function and its derivitive can be inserted into (A1) to give the density as a function of u. When y is given as a function of u, f(u), then approximations such as (4) give y as a funciton of u:

$$\overset{"}{y} = \{f(u)^{.14} - [1 - f(u)]^{.14}\}\ 4.91 \tag{A2}$$

Differentiating (A2) with respect to u gives:

$$\frac{d\overset{"}{y}}{du} = 0.6874\ \{f(u)^{-.86} + [1 - f(u)]^{-.86}\}\frac{df(u)}{du} \tag{A3}$$

Substituting (A2) and (A3) into (A1) gives the density as a function of u.

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